Application of Non-Overlapping Mortar Finite Element Method in Eddy Current Problem Involving Movement

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Abstract — This paper applied non-overlapping mortar finite element method (NO-MFEM) in eddy current problem involving movement. When deal with movement by NO-MFEM, the whole region is divided into two sub-regions, one contains the moving part and the other contains the source current. The two sub-regions were discretized independently and the two set of meshes are non-conforming on the interface. When the moving part changes location, it only needs to change the node coordinates in moving region. In this paper, the fundamental of NO-MFEM is introduced. The dynamic characteristics of an induction coil gun are analyzed by NO-MFEM. Numerical results obtained are compared with experimental data and results of other numerical methods, reasonable agreement is achieved and the efficiency of NO-MFEM is proved.

I. INTRODUCTION

Eddy current problem involving movement is always one of the most complex and difficult issues in computational electromagnetics [1]. The calculation of eddy current involving movement is very important in the analysis and design of some electro-mechanical devices, such as electric rotating machine, linear induction machine, maglev, electromagnetic launchers.

Because of the existence of moving parts, the management of grids in accordance with the relative movement of components in system is troublesome. The method of remeshing [2] at each time step is time-consuming and difficult to operate. The finite element and boundary element coupling method (FE-BECM)[3] can deal with arbitrary movement, but the introduction of boundary element matrix destroys the symmetric and positive definite of system matrix. Composite grid method (CGM)[4]-[5] is also a convenient method for arbitrary movement, but in every time step of CGM there are many iterative step to serially execute, so CGM is time consuming and difficult to parallel implementation. In this paper a new domain decomposition method — non-overlapping mortar finite element method (NO-MFEM) is introduced and applied in coil gun problem.

II. THEORY

Mortar element method (MEM)[6] is a new type of domain decomposition method, MEM introduces a discretized mortar space to approximate the original continuous function space. The continuity of DOFs across the non-conforming interface is ensured by the surface integration in a weak sense. In NO-MFEM the sub-region was meshed independently with traditional finite elements and the two sets of meshes are non-conforming on the interface, as shown in Fig. 1. The NO-MFEM can "mortar" the interface between sub-domains effectively.

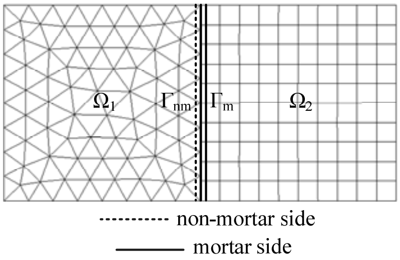


Fig. 1. Domain decomposition of NO-MFEM

The freedom $\boldsymbol{u}$ on the non-mortar side Γ_{nm} is function of $\boldsymbol{v}$ on mortar side Γ_m. The basis theory of the MEM interface continuous condition can be found in [6] and it may be written as

$$\boldsymbol{u} = \boldsymbol{Q}\boldsymbol{v} \tag{1}$$

where

$$\boldsymbol{Q}=\boldsymbol{C}^{-1}\boldsymbol{D} \tag{2}$$

The element in $\boldsymbol{C}$ and $\boldsymbol{D}$ are calculated by integral over interface

$$C(i,j)=\int_{\Gamma_{nm}} \psi_i \psi_j d\Gamma \tag{3}$$

$$D(i,j)=\int_{\Gamma_{nm}} \psi_i \varphi_j d\Gamma \tag{4}$$

The function ψ_i and φ_i are the base function of non-mortar side and mortar side. Denote vector of non-mortar nodes freedom in Ω_1 as $\boldsymbol{u}_{1\Gamma}$, vector of mortar nodes freedom in Ω_2 as $\boldsymbol{u}_{2\Gamma}$, vectors of rest nodes in the two sub-regions as $\boldsymbol{u}_{1i}$, $\boldsymbol{u}_{2i}$. According to (1), the vector of nodes in Fig. 1 can be written as

$$\begin{bmatrix} \boldsymbol{u}_{1\Gamma} \\ \boldsymbol{u}_{1i} \\ \hdashline \boldsymbol{u}_{2\Gamma} \\ \boldsymbol{u}_{2i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{Q} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \hdashline \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_{1i} \\ \hdashline \boldsymbol{u}_{2\Gamma} \\ \boldsymbol{u}_{2i} \end{bmatrix} = \tilde{\boldsymbol{Q}} \begin{bmatrix} \boldsymbol{u}_{1i} \\ \hdashline \boldsymbol{u}_{2\Gamma} \\ \boldsymbol{u}_{2i} \end{bmatrix} \tag{5}$$

So the non-mortar node freedom will not appear in system matrix of NO-MFEM.

The use of NO-MFEM in rotation problem is report in [7]-[8], but there is no report of its use in linear slide problem.

III. ANALYSIS OF COIL GUN MODEL

NO-MFEM is applied in the simulation of dynamic characteristics of coil gun. The model is taken from [9]. In the model, amarture is an aluminum cylinder and excitation in the coil is current impulse. The coil gun's dimensions are shown in Fig. 2.

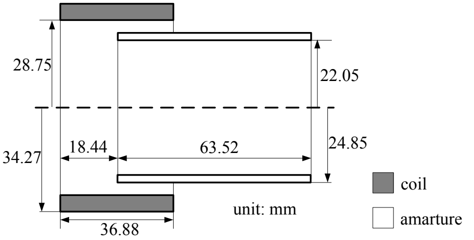
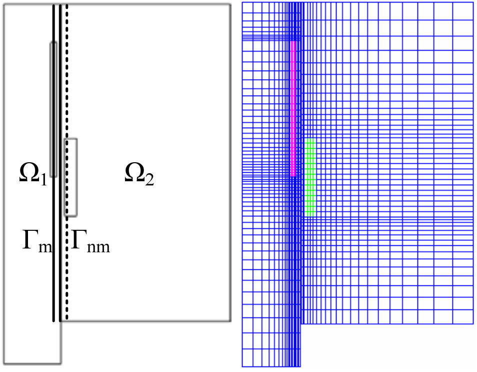


Fig. 2. The schematic drawing of coil gun model and its dimensions (mm)

Build 2D axisymmetric NO-MFEM model for the coil gun. The whole region is divided into two sub-regions; both the sub-regions are meshed with linear quadrilateral element, as shown in Fig. 3.



(a) (b)

Fig. 3. NO-MFEM model of coil gun (a. domain decomposition; b. discretization of model)

When the armature slides forward in every time step, it is only need to change nodes coordinate and information of mortar elements and mortar nodes in Ω_1. The exciting coil's current waveform is shown in Fig. 4.

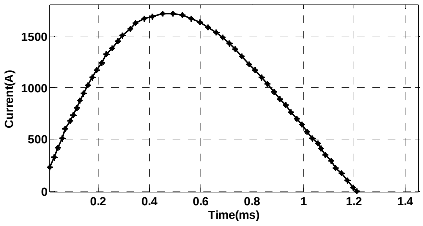
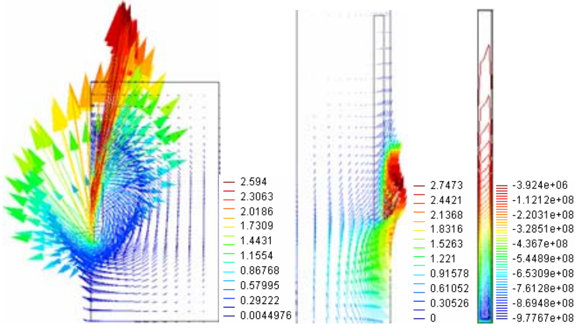


Fig. 4. Exciting current fed into the excitation coil

The field distribution at 0.3ms is shown in Fig. 5. Project's force curve and velocity curve are shown in Fig. 6 and Fig. 7 respectively. In Fig. 7, the results of 3D CGM [4], 3D FE-BECM [3], sliding meshes [9] and experiment [9] are also displayed.



(a) (b) (c)

Fig. 5. Field distribution at 0.3ms (a. flux density vector of exciting coil; b. flux density vector of armature; c. contour of eddy current in armature)

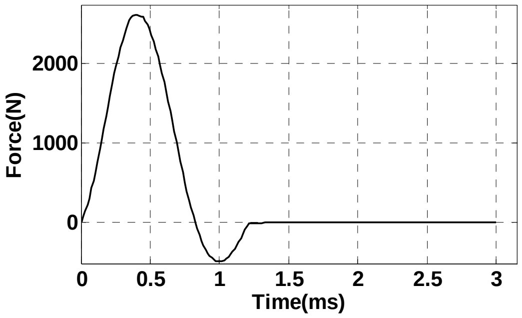


Fig. 6. Force wave of armature

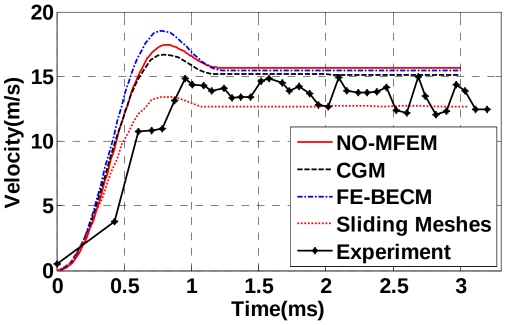


Fig. 7. Waveform comparison of armature velocity

IV. REFERENCES

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